Technische Universität München

Fakultät für Mathematik
Lehrstuhl für Mathematische
Statistik

Spatial copula day

Abstracts

Prof. Dr. Jürgen Pilz: Copula-based Bayesian spatial modeling

In this talk I will first briefly review some basic geostatistical concepts and their limitations and shortcomings. I will then continue presenting the efforts we have made in Klagenfurt in developing a copula-based approach to spatial modeling and interpolation that works with both continuous and discrete margins and which makes it possible to include geometric anisotropy and covariates, e.g. a spatial trend.
This model generalizes the concept of transformed Gaussian random fields and provides an alternative to Diggle and Ribeiro's (2007) approach on the basis of generalized linear geostatistical models. Moreover, this methodology can be easily incorporated in a Bayesian framework to additionally consider the uncertainty about the model parameters involved. In case of discrete univariate marginals exact maximum likelihood estimation is computationally intractable. To circumvent the computational burden we propose two methods for approximate inference. The first is a composite likelihood approach where the estimates maximize the pairwise likelihood. A second possibility for approximate inference is to use the so-called generalized quantile transform.
We will demonstrate our approaches with a with two real data examples, using spatialCopula, a MATLAB toolbox for copula-based spatial analysis developed by Kazianka (2012).

Annette Möller: Multivariate and spatial probabilistic weather forecasting using Gaussian copulas

Numerical Weather Prediction (NWP) uses physical models to derive predictions of future weather conditions. Statistical postprocessing methods construct predictive distributions from ensembles of NWP outputs to correct for biases and imperfect representations of the forecast uncertainty.
Many established postprocessing methods focus on a single weather quantity at a given location, ignoring possible interactions between weather quantities and the fact that forecast errors often exhibit spatial structures. This work uses Gaussian copulas to extend existing postprocessing techniques and to acknowledge that weather patterns display relationships between variables as well as spatial dependencies.
We employ the univariate postprocessing method Bayesian Model Averaging (BMA) to estimate the marginal distributions of different types of weather quantities. These are then combined in a Gaussian copula to obtain a multivariate distribution. Furthermore we extend the Ensemble Model Output Statistics (EMOS) method to spatially adaptive parameters by assuming a spatial Gaussian field on the model parameters. A new method, based on the fact that Gaussian fields with

Matérn covariance function are solutions to a certain stochastic partial differential equation, yields computational benefit by using a Markovian approximation of the Gaussian field. As this method only provides spatial marginal predictive distributions, the marginals are combined with a spatial correlation matrix in a Gaussian copula approach to obtain spatially correlated joint predictive samples.

Roman Schefzik: Physically coherent probabilistic weather forecasts via ensemble copula coupling (ECC)

State of the art weather forecasts depend on ensemble prediction systems, which consist of multiple runs of dynamical numerical weather prediction models differing in the initial conditions and/or the parameterized numerical representation of the atmosphere. Statistical postprocessing of the ensemble forecasts is required to realize their full potential, in the sense that biases and dispersion errors need to be addressed. Current postprocessing approaches mostly apply to a single weather quantity at a single location and for a single prediction horizon only. However, in many applications there is a critical need to account for spatial, temporal and inter-variable dependencies.
To address this, we propose a tool called ensemble copula coupling (ECC), in which existing univariate postprocessing methods are employed to obtain calibrated and sharp forecasts for each location, variable and look-ahead time separately. Then, the univariate distributions are aggregated in a discrete copula approach. The key idea is that the postprocessed ECC ensemble inherits the multivariate rank dependence pattern from the unprocessed ensemble, thereby capturing the flow dependence.
In this talk, we present the ECC approach, study its relationships to discrete copulas, and assess the predictive performance in an application to ensemble data over Germany.

Ulf Schepsmeier: Spatial R-vine copula models

R-vine copulas are a very flexible class of multivariate copulas based on a pair-copula construction (PCC). The main idea is to decompose a multivariate density into bivariate copulas and marginal densities which can be modeled independently. Such a decomposition is not unique. Nested trees allow to identify the decomposition of a d-dimensional density into d(d-1)/2 bivariate copulas. They also characterize the dependence structure.
The fitting of an R-vine copula is devided into three parts: Tree structure selection for the decomposition, copula family selection and copula parameter estimation. While the second and third step are sufficiently studied and adequate procedures are available, for the first step little work is done. Classical R-vine structure selection methods are based on (rank-based) correlation measures as edge weights. Usually pair-wise Kendall's taus are estimated and are used in a maximum spanning tree algorithm to capture strong dependencies (Dissmann et al., 2011). Spatial R-vine copula models extend the class of R-vine models by using additional spatial information. Pair-wise distances can be used as edge weights in the tree structure

selection procedure. The challenge is the modeling of the higher order trees representing conditional pair-copulas. We will present two possible approaches.
In this talk we will introduce the R-vine copulas models, its properties, estimation methods and selection algorithms. Further we will present its extension to spatial data and apply the introduced models to climate date.

Benedikt Gräler: An Application of Vine Copulas in the Spatio-Temporal Domain

Many natural phenomena are observed at a discrete set of locations in space and time. The underlying process is commonly modeled as a spatio-temporal random process $Z(s, t)$ over some spatio-temporal domain $S \times T$. Extending spatial random fields to spatio-temporal random fields typically introduces a complex spatio-temporal dependence structure. Due to their high degree of exibility, vine copulas can be used to model these relations. A good understanding of the spatio-temporal dependencies for a sample of a given spatio-temporal random field is, for instance, necessary to predict the random field at unobserved locations in space and time. In this context, a copula's dimension depends on the number of points involved in a local neighbourhood. It is desirable to build these multidimensional copulas out of bivariate ones, as those are quite well understood and are easy to estimate.
We adapted vine copulas to the spatio-temporal domain by introducing a distance dependent *bivariate spatio-temporal copula* in the first tree of the vine copula. Our approach solves two constraints of recent geostatistical models using copulas. At first, a vine copula is not limited to a single family and, secondly, it is not limited to copulas that are capable of describing the whole range from high dependencies to independence. In general, any bivariate copula family can be considered for different spatio-temporal lag classes and the copula families may change over space and time.
The asymmetric dependencies typically present in the temporal domain can easily be captured incorporating asymmetric copulas. Thus, the vine copula is capable of capturing spatial and temporal dependencies of the process. Predictions of the mean, median or given quantiles can be inferred from the distribution calculated by conditioning the spatio-temporal copula on the values within the neighbourhood of the unobserved location in space and time. The spatial, temporal and spatio-temporal distances will adjust the set of copula parameters. Spatio-temporal vine copulas enable us to model random fields over space and time in a very exible way.

Eike Brechmann: Air pollution modeling at different stations using a hierarchical copula construction

Due to its negative health effects air pollution today is a major concern. To better understand the joint behavior of pollutants we therefore propose a hierarchical copula construction which explicitly allows for spatial effects between sites and accounts for meteorological effects on air pollution by marginal regression models. The model is based on a so-called hierarchical Kendall copula, which aggregates

dependence information of groups of variables in different hierarchical levels using the Kendall distribution function, the multivariate analog of the probability integral transform. Hierarchical Kendall copulas are therefore able to model complex dependence patterns without severe restrictions.
As an illustration of the proposed model we analyze PM10, NO2 and CO time series from seven cities in south-western California.

Christine Steinkohl: Max-stable processes for extremes observed in space and time with application to radar rainfall measurements

Max-stable processes have proved to be useful for the statistical modelling of spatial extremes. Several representations of max-stable random fields have been proposed in the literature. One such representation is based on a limit of normalized and scaled pointwise maxima of stationary Gaussian processes that was first introduced by Kabluchko, Schlather and de Haan (2009).
We present statistical inference for max-stable space-time processes that are defined in an analogous fashion. We first describe pairwise likelihood estimation, where the pairwise density of the process is used to estimate the model parameters. In a second step we present an alternative semiparametric estimation procedure for the parameters which is based on a non-parametric estimate of the extremogram. Finally, the introduced model and methods are applied to radar rainfall measurements in order to quantify the extremal properties of the space-time observations. (This is joint work with Richard A. Davis and Claudia Klüppelberg.)